

# Evaluation of self-organizing systems using quantitative measures

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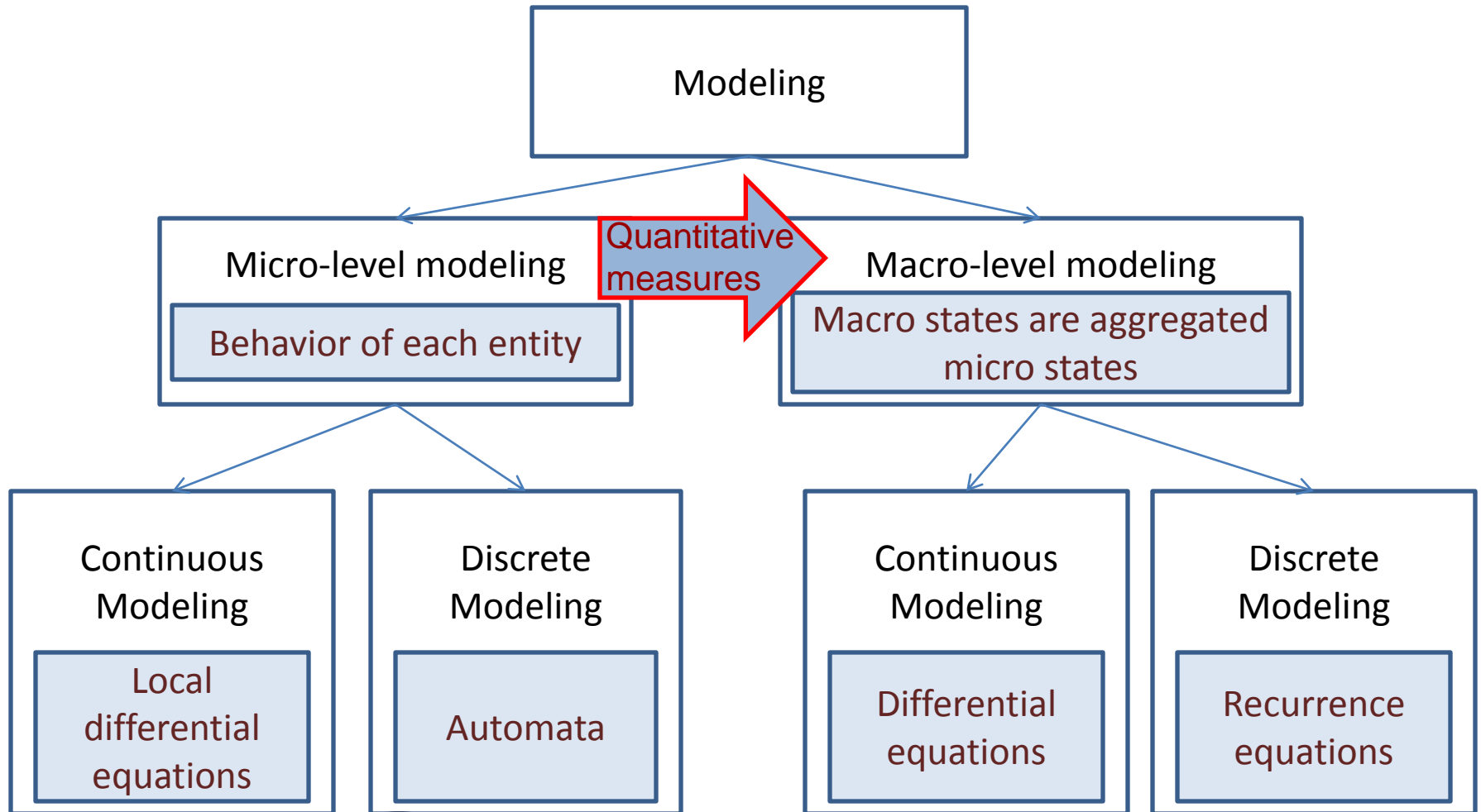
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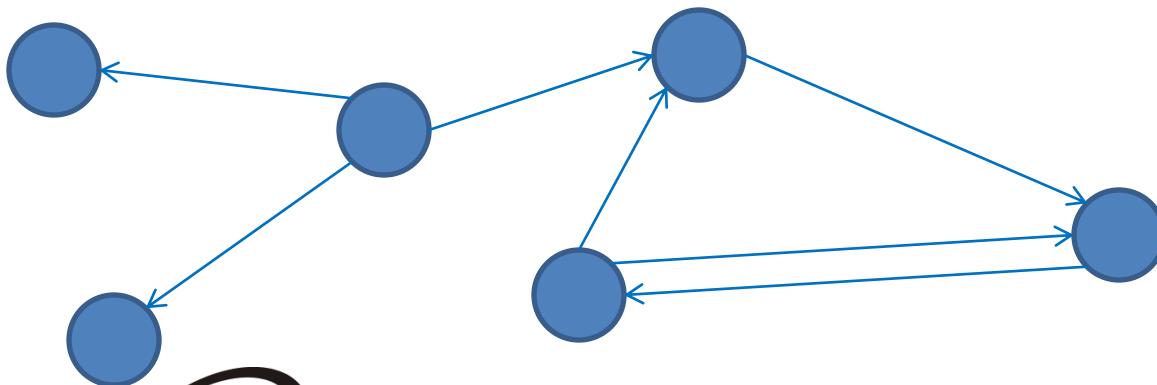
1. Modeling of systems
2. Quantitative measures
3. Approximation methods
4. Example: Synchronization of sensor nodes
5. Example: Evacuation



# Modeling approaches



- **Topology** can be described by a **directed graph**  $G = (V, K)$
- **Behavior** can be described by local rules for each node



# From Micro-level to Macro-level: Quantitative Measures

To measure **self-organizing properties** of a system, we need to determine the quantity of information in the system.

Statistical **Entropy** of a random variable  $X$ :

$$H(X) = - \sum P(X=w) \log_2 P(X=w)$$

With this concept we can measure for each point of time

- the information in the whole **system**
- the information on the **internal edges**
- the information on the **input edges**
- the information on the **control edges**
- the information on the **output edges**



**Quantitative measures** can be obtained (partly based on information entropy):

Levels of

- **emergence**
  - How many globally coherent patterns are induced by local interactions?
- **autonomy**
  - How much control data from external entities are needed to keep the system running?
- **target orientation**
  - Is the high level goal, that the system designer had in his mind, reached by the system?
- **adaptivity**
  - Is the high level goal still reached after changes in the environment?
- **resilience**
  - Is the high level goal still reached after unexpected impacts on the system (e.g. break down of nodes, attacks by an intruder, ...)?
- **homogeneity**
  - Do all nodes have the same behavior?
- **global state awareness**
  - How much information does a single node have about the global state?



To measure the **level of emergence**

$$\varepsilon \in [0, 1]$$

of a system, we compute the dependencies between the values on the edges  $k \in K$ .

At time  $t$  we compare the information contained in all edges with the sum of the information contained in each single edge:

$$\varepsilon_t = 1 - (H(\text{Conf}_t|_K) / \sum_{k \in K} H(\text{Conf}_t|_k))$$

where  $\text{Conf}_t$  is the global state containing all local states and all values on the edges.

Level of **emergence** of the whole system  $S$ :

$$\varepsilon(S) = \text{Avg}(t \mapsto \varepsilon_t), \quad \text{where Avg is the average value of the map}$$

$\varepsilon \approx 1$     high level of emergence    (many dependencies)

$\varepsilon \approx 0$     low level of emergence    (few dependencies)



Before a new system is designed, we have a **goal** of the system in our mind:  
The system should fulfill a given purpose.

In the model, the goal can be described by a **valuation** of configurations:  
 $b : \text{Conf} \rightarrow [0, 1]$  (Conf is the set of all global states)

Level of **target orientation** at time  $t$ :

$$TO_t = \mathbf{E}(b(\text{Conf}_t)), \quad \text{where } \mathbf{E} \text{ is the mean value of the random variable}$$

Level of **target orientation** of the whole system  $S$ :

$$TO(S) = \text{Avg}(t \mapsto TO_t)$$

$TO(S) \approx 1$  means that the system runs through many good configurations  
 $\Rightarrow$  high level of target orientation

$TO(S) \approx 0$  means that the system runs through many bad configurations  
 $\Rightarrow$  low level of target orientation





## ■ Problem: Scalability

- In large systems, it might be difficult to compute the quantitative measures.
- The global state space (the set of all configurations) grows exponentially with the number of the entities in the system.
- To be able to analyze global properties of the system, entropies of global random variables like  $H(\text{Conf}_t)$  are difficult to calculate analytically.
- Therefore we need methods to approximate the needed values.



- **Simulation runs** can be used to approximate probabilities and entropies.
- $R$  : Number of simulation runs.
- Each simulation run leads to a **time series**, which is the configuration sequence  $c_0 \rightarrow c_1 \rightarrow c_2 \rightarrow \dots$  produced by the simulation run.
- The **probability** of a value  $a \in A$  on a single edge  $k \in K$  at time  $t > 0$  can be approximated by the **relative frequency**  $\text{rel}_{t,k,a}$  by counting the number of time series, which contains the value  $a$  on the edge  $k$  at time  $t$ .
- For **probabilities of global valuations** like  $P(\text{Conf}_t = c)$  or  $P(\text{Conf}_t|_K = a)$  the **range of values** for these random variables is too large, so the relative frequencies received from simulation runs are too inaccurate for the approximation of probabilities.
- For such global probabilities, we investigate **different approximation methods**.



## Classification

- The probability space is divided into different **classes**.
- The **relative frequency** for each class is calculated from the time series.
- The probability for a single element of the class can be approximated by the **relative frequency divided by the size of the class**.
- For  $P(\text{Conf}_t|_K = a)$  the classification can be done by choosing a **subset**  $K_0 \subseteq K$  and build  $|A|^{|K_0|}$  equivalence classes  $[b] := \{ a \in A^K \mid a|_{K_0} = b \}$  for  $b \in A^{K_0}$ , where the size of each class is  $|[b]| = |A|^{|K \setminus K_0|}$ .
- Now the **relative frequencies**  $\text{rel}_{t, K_0, b}$  are calculated for each class  $[b]$  and the probability  $P(\text{Conf}_t|_K = a)$  is approximated by

$$P(\text{Conf}_t|_K = a) \approx 1/|[b]| \cdot \text{rel}_{t, K_0, b} \quad \text{for } b = a|_{K_0}$$



## Classification

- Example:
  - $A = \{ 0, 1 \}$
  - $K_0 = \{ k \}$
  - Then we have two equivalence classes of configurations:
    - In the first class are all configurations  $c$  with  $c_k(k) = 0$
    - In the second class are all configurations  $c$  with  $c_k(k) = 1$
- After calculating the approximations of the probabilities, we get an **approximation of the quantitative measure.**



## Parzen window approach

- For a random variable  $X$  with a sample set  $W = \{ w_1, \dots, w_R \}$  (observations of  $X$ ) we can use the kernel density estimator based on a **Gaussian kernel**

$$p(a) = \frac{1}{R} \sum_{j=1}^R \frac{1}{(2\pi h^2)^{dim/2}} \exp\left(-\frac{1}{2} \frac{dist(a, w_j)^2}{h^2}\right)$$

with

$dim$  = dimension of the random variable  $X$

$a \in \mathbb{R}^{dim}$

$p(a)$  : approximation of the density of  $X$  at  $a$

$R$  : number of samples for  $X$

$dist(a, w_j)$  : Euclidean distance between  $a$  and  $w_j$

$h$  : user-defined parameter for changing variance and bias



## Parzen window approach

- By integrating over the density function  $p$ , we can calculate **probabilities** for the random variable  $X$ .
- In our case, we consider discrete systems, so the random variables (e.g.  $\text{Conf}_t$ ) are **discrete**.
- If we assume that the random variable only yields **integer values** for each component, then we can use  $P(X = c) = P(\text{dist}_\infty(X, c) \leq \frac{1}{2})$  for the approximation, where  $\text{dist}_\infty(a, b) = \max\{ |a_i - b_i| : i = 1, 2, \dots, \text{dim} \}$  is the distance induced by the maximum norm.
- Since the set  $\{ a \mid \text{dist}_\infty(a, c) \leq \frac{1}{2} \}$  is a hypercube of size 1, the value  $P(\text{dist}_\infty(X, c) \leq \frac{1}{2})$  can be approximated directly with the **density function**  $p$ , i.e.  $P(X = c) \approx p(c)$ .
- This approximation can then be used to get approximations of the entropies  $H(\text{Conf}_t)$  and  $H(\text{Conf}_t | \mathcal{K})$ .



## Restriction of the set of initial configurations

- When we have a system, in which large parts are **deterministic**, then a restriction of the set of the initial configurations reduces the complexity.
- Let  $\Gamma_0 \subseteq \Gamma$  be a set of **initial configurations**.
- Then the **time series** are received from simulation runs starting in  $\Gamma_0$ .
- If all automata are **deterministic**, two simulation runs with the same initial configuration would lead to the same time series, so for each initial configuration  $c_0$  at most one simulation run is needed.
- If some automata are **stochastic**, different initial configurations might lead to different time series.



## Restriction of the set of initial configurations

- The entropy  $H(\text{Conf}_t |_K)$  (and analogously  $H(\text{Conf}_t)$ ) can then be derived by using the **relative frequency** as an approximation for the probability  $P(\text{Conf}_t |_K = \underline{a})$ :

$$\begin{aligned} H(\text{Conf}_t |_K) &= - \sum_{\underline{a} \in A^K} P(\text{Conf}_t |_K = \underline{a}) \log_2 P(\text{Conf}_t |_K = \underline{a}) \\ &\approx - \sum_{\underline{a} \in A^K} \text{rel}_{t,\underline{a}} \log_2 \text{rel}_{t,\underline{a}} \end{aligned}$$





- All these approximation methods are based on **time series**.
- By using **simulation runs**, time series consisting of the configuration sequences can be obtained.
- Instead of calculating the measures analytically in the model, it is possible to get approximations of the measures directly from the set of **time series**.
- Since the model is not needed anymore, this can be **generalized to arbitrary time series** of configurations: For each set of configuration sequences, the quantitative measures, which were defined analytically only for the model, can be approximated by considering only the time series.
- This allows the usage of **experimental data** from the real world without the need of the model: By measuring the parameters of interest in the real world system, we get some time series, which can be used for the calculation of the quantitative measures.



# Example:

# Slot synchronizing in wireless networks

## Wireless network:



- For communication, time is divided into slots.
- There is no central clock, which defines when a slot begins.
- The nodes try to synchronize the slots.

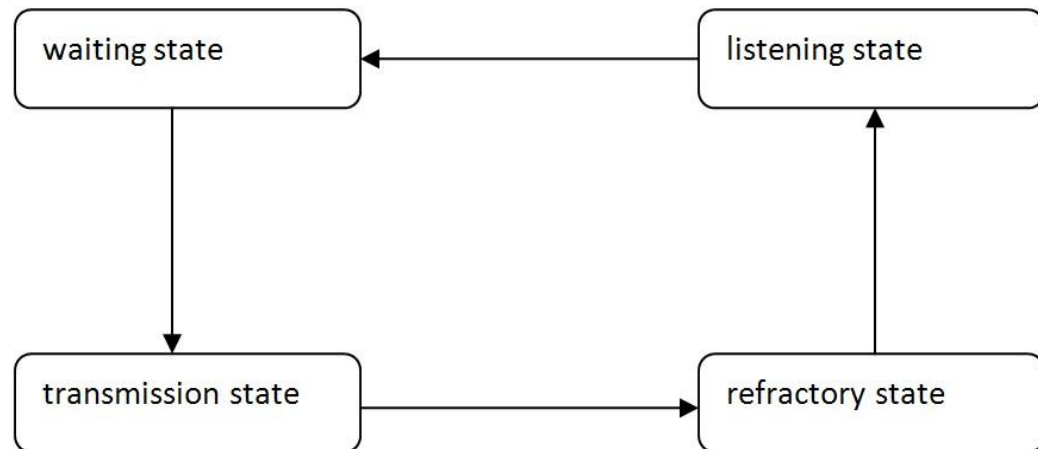


# Example:

## Slot synchronizing in wireless networks

### Slot synchronization algorithm of Tyrrell, Auer and Bettstetter:

- At each point of time, each node is in one of four different states:
- In the **transmission state**, the node transmits a pulse to it's neighbors to indicate the beginning of a slot.
- In the **listening state**, the node can receive and decode pulses from it's neighbors and it adjusts its phase function  $\phi$  according to these pulses. The listening state ends, when the threshold  $\phi_{\max} = 1$  is reached.
- In the **waiting state** and in the **refractory state**, the node does nothing.
- The length of an uncoupled cycle is  $2T$  with  $T > 0$ .



# Example:

# Slot synchronizing in wireless networks

## Simulation results show:

- **Two groups of synchronizations** are built.
- Inside each group we have good synchronization: Each object of the group fires a pulse at nearly the same time like the other objects of the group.
- The second group fires  $T$  time units after the first group.
- By using slots of length  $T$  we get a good slot synchronization



Calculation of the level of **target orientation**:

The **goal** is to minimize the time differences between the beginning of the slots of the nodes.

$\text{dist}_c(v,w)$ : **Slot distance** of nodes  $v, w \in V$  in configuration  $c$

$$b(c) = 1 - \frac{\sum_{v,w \in V} \text{dist}_c(v,w)}{|V^2| \cdot T/2}$$

After fixing the system parameters, we can approximate the level of target orientation using simulation runs.

$|V| = 30, T = 100, T_{\text{dec}} = 15, T_{\text{Tx}} = 45, T_{\text{refr}} = 35, T_{\text{wait}} = 40, \alpha = 1.2, \beta = 0.01,$   
 $R = 300, t = 1000$

**Target orientation:**  $TO_t(S, \Gamma) \approx \frac{1}{R} \sum_{j=1}^R b(\text{Conf}_{t,j}) \approx 0.996$



Level of **Emergence**:

$$\varepsilon_t = 1 - (H(\text{Conf}_t|_K) / \sum_{k \in K} H(\text{Conf}_t|_k))$$

The values  $H(\text{Conf}_t|_k)$  can be approximated by the **relative frequencies**.

For  $H(\text{Conf}_t|_K)$  we have different methods for **approximation**.



Level of **Emergence**:

**Classification:**

$$H(\text{Conf}_t|_K) \approx - \sum_{[b]\text{class}} \text{rel}_{t,K_0,[b]} \log_2 \left( \frac{1}{|[b]|} \cdot \text{rel}_{t,K_0,[b]} \right)$$

The result depends on the size of the classes.

$ K_0 $	870	860	700
$ [b] $	1	$2^{10}$	$2^{170}$
$\varepsilon_t(\mathcal{S}, \Gamma)$	0.988	0.980	0.773



Level of **Emergence**:

**Parzen Window:**

$$H(\text{Conf}_t|_k) \approx p(a) = \frac{1}{R} \sum_{j=1}^R \frac{1}{(2\pi h^2)^{\dim/2}} \exp\left(-\frac{1}{2} \frac{\text{dist}(a, w_j)^2}{h^2}\right)$$

The result depends on the user defined parameter  $h$  for the variance and bias:

$h$	0.478	0.479	0.4795	0.48
$\varepsilon_t(\mathcal{S}, \Gamma)$	0.778	0.956	0.981	0.991





Level of **Emergence**:

**Restriction of initial configurations:**

$$H(\text{Conf}_t|_K) \approx - \sum_{\underline{a} \in A^K} \text{rel}_{t,\underline{a}} \log_2 \text{rel}_{t,\underline{a}}$$

We set the number of simulation runs equal to the number of initial configurations.

$ I_0  = R$	10	1000	2000
$\varepsilon_t(\mathcal{S}, I)$	0.99	0.984	0.982



- It is planned to test the approximation methods for two applications of the project **SOCIONICAL** funded by the EU.
- SOCIONICAL focuses on the specific example of **Ambient Intelligence (Aml)** based smart environments.
- A key component of such environments is the ability to monitor user actions and to **adjust its configuration** and functionality accordingly.
- Thus, the system reacts to human behavior while at the same influencing it.
- The project will study global properties and emergent phenomena that arise in Aml based socio-technical systems from such local feedback loops and their coupling on two concrete **scenarios**:
  - emergency scenario
  - traffic scenario



# Example: Evacuation scenario

- **Scenario:** Evacuation in a building
  - Each person wears a life belt:
    - Ambient Intelligence (Aml) device, which is able to communicate with other life belts to improve the evacuation.



## ■ Topology

- Each node in the graph represents one person wearing a life belt.
- Each edge in the graph represents a communication channel
- Since the persons move around, the topology changes during time
  - $\Rightarrow$  Graph  $G_t = (V, E_t)$  depends on time  $t$
- In a simple scenario, external nodes are not needed. The system will self-organize.
  - In a more complex scenario, external nodes can be introduced to model changes in the environment (e.g. break down of a part of the building, etc.)



# Example: Evacuation scenario

For the **target orientation** we need a **valuation** of configurations:

$b : \text{Conf} \rightarrow [0, 1]$

- In the evacuation scenario the good configurations are those where many people have already escaped:
  - $b(c) = \text{\#escaped}/N$   $N = \text{number of persons}$
- Consider a run of the system starting at time  $t = 0$  ending at  $t = T$ .
- $TO_t = E(b(\text{Conf}_t))$  is a nondecreasing function
- **Goal:** Try to maximize  $TO_T$
- Different rule sets can be compared.
- Rules with different parameters can be compared.



# Example: Traffic scenario

## ■ Scenario:

- There was an **accident** on a highway
- Other cars **slow down**, when they are approaching the scene of accident.
- Possibility for **Aml device**:
  - Some cars may contain a device, which **send information** (e.g. velocity, acceleration, etc.) to other cars to improve safety.
  - The device can give **hints to the driver** (e.g. slow down) before he reaches the scene of accident
- **Question:** How does the Aml device change the traffic flow around the scene of accident?



# Example: Traffic scenario

- **Level of target orientation**
  - From the point of view of the drivers, it would be good, if the **variance** of the velocities is low.
  - Define **good configurations** by the valuation
    - $b(c) = \text{Var}_{\text{normal}}(v) / \text{Var}(v)$ 
      - where  $\text{Var}_{\text{normal}}(v)$  is the **variance** of velocity in the system without accident (can be calculated analytically, so no simulation runs are needed)
    - Then the **level of target orientation** can be approximated by using the time series received from the simulation runs.



# Example: Traffic scenario

- **Level of emergence**
  - Can the traffic jam induced by the accident be seen as an emergent pattern of the system?





- For the analysis with quantitative measures in the **evacuation scenario**, the University of Halle-Wittenberg is currently adapting a simulation software to produce the necessary time series.
- For the analysis with quantitative measures in the **traffic scenario**, the University of Munich is currently adapting a simulation software to produce the necessary time series.
- University of Passau is currently working on a software for the **evaluation of the quantitative measures** for the time series of simulation runs.



The mathematical modeling can be used for **a wide variety of systems:**

- **Technical systems**
  - **Biological systems**
  - **Physical systems**
- and many more.

The models can help to analyse the behavior of complex systems.

**Quantitative measures** provide a link from the micro level to the macro level:

- They describe **global properties of the system**
- They can be used for the **analysis** of real world systems.
- They can be used for **design, engineering** and **optimization** of new systems.



**Thank you for your attention**

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